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ABSTRACT

Although mathematics can be described as a formalized language, the signs and symbols of mathematical representation are subject to individual interpretation, which, if inaccurate, can lead to inhibitions about and interferences with the mathematical learning process. The purpose of this study was to explore communication in an intermediate-level college discrete mathematics classroom and to elucidate possible differences in interpretation that could detract from the attainment of an accurate shared meaning, as well as the effects of those differences upon achievement levels. Transcripts of multiple interviews with 12 purposively chosen student volunteers were analyzed for discrepant, imitative, or interpretive analyses of the course material, and comparisons were made between students' test results and the various types of students' explanations. When students' test scores and final interview ratings were examined, no significant patterns were noted beyond the fact that high achieving students differed little from the other students in terms of frequency of interpretative answers to interview questions. (JJK)

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Semiotics in the College Mathematics Classroom

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Abstract

This paper focused on students' understandings of lectures in an intermediate college mathematics class, Finite Mathematics. Segments of the videotaped class focusing on symbolic notation in the lecture were replayed to the instructor and to students individually. They were each interviewed and asked their explanation of the material being presented. Transcripts of the explanation interviews were analyzed for discrepant, imitative, or interpretive analyses of the material that was presented. Finally, test results over the material were compared with types of explanation.

Semiotics in the College Mathematics Classroom

> At 10:07, three minutes before the scheduled start of class, Laura, the instructor, arrives. She smiles at the scattering of students and talks a little about the weather and her walk across campus. A few students approach her to ask about the recent test, a class date they will miss... At 10:10 the class begins; about fifty of the original 90 students who were enrolled ten weeks ago are present today for the introduction of new material on Markov chains. At 10:34, Lorrie, one of two African American students in the study, arrives. Paola, a Hispanic student who is also in the study, is among those who miss today's class.

Recent reports on the status of education in the United States have called attention to curriculum at the elementary, secondary, and the postsecondary levels. In particular, of sixteen major reports published since 1983, nearly all focused on the importance of mathematics and mathematics achievement to the country's future (Shane, 1987). Many students who come to college have insufficient mathematics preparation at the secondary level. Thus most colleges offer courses to prepare students for college level work.

The success rate of students in these classes is typically 50% or less. A disproportionately high percentage of women and minorities fail such courses (Hackett, 1985; Melange, 1987). Unfortunately, these mathematics courses serve as educational and vocational "gate keepers." A student who is unsuccessful at mastering mathematics skills looses the opportunity to enroll in a broad range of college courses. Thus their choice of career is

limited.

While many expressed concern about difficulties college students have with mathematics, there is little research on the mathematics experience at the postsecondary level. Although there has been a very specific focus on students who have what may be termed "math anxiety" (Stodolsky, 1985; Williams, 1988), little research has been conducted on learning of mathematics in the college classroom. Additionally, most studies of the college classroom tend to focus on quantitative relationships between evaluations of the classroom environment and students' grades, ignoring finer nuances of transmission of meaning and actual learning. Mentowski and Chickering (1987) have called for a higher education research agenda that places greater emphasis on learning in the college classroom.

Communication in American society is dominated by mathematical and verbal symbols. However, the signs and symbols that are used to communicate about mathematics are subject to individual interpretation (Cunningham, in press; Eco, 1976). Indeed, mathematics may be described as a formalized language (Eco, 1976). It is possible that misinterpretation of signs (written, verbal, and nonverbal) in a mathematics classroom may lead to mistaken notions regarding the nature of mathematics. Such inaccuracies about the nature of a domain of knowledge can inhibit or interfere with learning (such as incorrect knowledge about the nature of mathematics) (Alexander & Judy, 1988).

Symbols commonly employed by instructors to communicate in

mathematics classes may be misunderstood by students in those classes. Such differences in linguistic interpretations have been shown to cause problems for language minority students in mathematics classes (Cocking & Mestre, 1988). It is possible that such differences can cause problems for other students as well.

The purpose of this study was to explore communication in an intermediate level college mathematics classroom. The following questions guided the study. What are the signs that are used to communicate in the classroom? How does the instructor of the class interpret those signs? How do students in the class interpret those signs? Do differences in interpretation of signs in a mathematics classroom vary by success levels of students? Which students are more likely to share the instructor's interpretation of those signs?

Method

The primary data collection technique employed was similar to Guba and Lincoln's hermeneutic dialectic (Guba, 1988) and Glazer and Strauss' (1967) constant comparative method. However, the purpose of this study was not to negotiate consensus or shared meaning; rather, the purpose was to elucidate differences in interpretation (dicensus) that could inhibit the achievement of shared meaning.

The Setting

The setting was a college level mathematics class entitled

"Finite Mathematics" at a midwestern research university. The class included business as well as social science majors, many of whom had already successfully completed one semester of calculus. It fulfilled general university distribution requirements for mathematics. Additionally, it was an alternative choice to second semester calculus for many of the business majors in the class.

One of four major class segments, covering material on Markov Chains, was chosen for this study. Based on the instructor's knowledge, it was decided that this was material with which students would have had the least previous exposure. Other segments of the class included topics on probability, statistics, systems of equations, matrix algebra, and linear programming.

Markov chains are stochastic processes (one definition of stochastic process is "An experiment which consists of a sequence of subexperiments" (Maki & Thompson, 1989:99). A stochastic process is called a Markov chain if 1) it includes a fixed number of states, and at each stage the process is in one of those states, and 2) the conditional probability of a transition from any given state to any other state depends only on the two states and not on the preceding transitions (Maki & Thompson, 1989).

As an example of a Markov chain, consider a basketball player shooting free throws. If she makes a free throw, the probability that she makes the next one is .7, the probability

that she misses is .3. If she misses a free throw, the probability that she makes the next one is .6 and that she misses the next one also, .4. This Markov chain has possible two states, make (1) or miss (2). Information about this Markov chain may be represented symbolically in one of three ways:

a) By listing the probabilities:

P_{12} = .3 (probability of going from make to miss)

P_{11} = .7 (probability of going from make to make)

P_{21} = .6 (probability of going from miss to make)

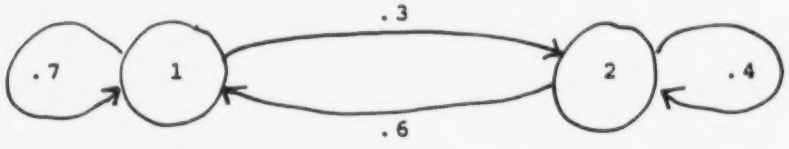
P_{22} = .4 (probability of going from miss to miss)

b) By creating a transition matrix:

$$P = \begin{bmatrix} .7 & .3 \\ .6 & .4 \end{bmatrix}$$

(The first element in the second row indicates the probability (.6) of going from miss to make.)

c) By drawing a transition diagram:



This seemingly heavy use of notation is not unusual in a mathematics classroom. Thus, mathematics affords a unique opportunity to study students' interpretations of signs.

<u>The Instructor</u>

The instructor, Laura, holds two master's degrees, one in

mathematics, and one in mathematics education. She has fifteen year's experience teaching college level mathematics. She has authored several mathematics textbooks and teacher's guides. Additionally, she has won several awards for outstanding teaching in the mathematics classroom. For the past eight years she has been the coordinator of the mathematics learning skills program as well as a lecturer in mathematics at the university under study.

Researcher as Instrument

The researcher holds a master's degree in mathematics and a Ph.D in education and has seven years' experience teaching college level courses. The author has many publications focusing on a variety of college student outcomes and research methods. Recent publications focus on success in low level college mathematics classrooms.

Sample

The researcher visited a class session and explained the purpose and procedures for the research project. Students who volunteered were given a form to fill out. The instructor and the researcher used purposive sampling to select 12 students from the volunteers to participate in the project. Respondents were chosen to include male and female as well as culturally diverse students at various levels of achievement within the class. The sample included 7 women and 5 men. Two were African American and one was Hispanic American. Students were selected from the top, middle, and low ranges of achievement in the class.

At the end of the term, final grades earned by the students included, one A, one A-, two B+s, one B, one B-, two Cs, two C-s, one D+, and one D-.

Procedure

During the first week of classes of the spring semester demographic data and information regarding students' high school exposure to mathematics were collected by the instructor. Additionally, scores on the Mathematics Skills Assessment Test and an arithmetic pretest are coded on students' records. (This information is collected in a standardized way for all low and intermediate level mathematics classes).

During the semester, four class lectures within the "Markov chain" segment of the class were videotaped. Simultaneously, the researcher viewed the class noting the instructor's as well as students' uses of signs in the classroom (writing on the board, verbal explanation, verbal cues, nonverbal cues).

Immediately following the class, the instructor and the researcher viewed portions of the videotape. The researcher asked the instructor questions about the signs that were employed in the just-completed class session. The questions were simplistic and were intended to be of a type that students in the class might also be able to answer. The same day, half of the students were interviewed using the same questions. During the interview students were shown the same segments of the tape and asked their interpretation of the signs being employed by the instructor. All interviews were recorded.

Following the next class session, the remaining half of the sample was interviewed following the procedures described above. Each student participated in interviews for two class sessions (one introductory material and a second higher level material) for a total of 24 student interviews. All interviews were transcribed and are currently being analyzed. The analysis will be compared with students' achievement in the class.

Data Analysis

Data analysis began with the first interview. This is consistent with the constant comparative method described by Glaser and Strauss (1967). As the interviews were conducted, hypotheses were formed that were then researched in later interviews. For example, it became apparent during the first round of interviews that a hypothesis set forth initially (high achieving students' explanations of signs would most closely resemble the instructor's explanations) seemed to be too simplistic. During the second round of interviews, an alternative hypothesis, that high achieving students put their own interpretations on explanations of mathematic phenomena, was explored.

Following the interviews, the audiotapes were transcribed to hard copy. In an initial reading of the transcripts, codes were made in the margins to identify types of symbols used in the class and to categorize student responses. The explanations were initially coded as "wrong", "as an 'I don't know,'" as "imitations" of the instructor's explanations, or as

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"interpretation" that went beyond the instructor's own explanations. During a second round of coding, impressions were refined and examples were identified for types of symbols that were being used in the classroom.

Establishing Trustworthiness

Lincoln and Guba's (1985) criteria for establishing trustworthiness in naturalistic inquiry guided this study. Credibility was established through the use of prolonged and persistent engagement, triangulation, peer debriefing, and member checks. One instructor and twelve students were engaged in 26 interviews to elicit their personal explanations of the material being presented in class. Triangulation was employed through observations of four classes (two on introductory material, two advanced material), the instructor's and students' personal explanations of the class, and the use of multiple respondents.

The student sample as well as the instructor conducted "member checks." The instructor and the interviewed students were sent copies of the transcripts of their own interviews as well as copies of the preliminary results. They were asked to contact the researcher with any questions, comments, or challenges to the material as presented.

The instructor also served as peer debriefer for the study. She continually served as a person to bounce ideas off during the data collection. She was asked to read and respond to the final report.

For transferability, attempts were made to be as accurate as possible about the content of the course, the classroom setting, and the skill level of the students involved. Thus the reader should be able to judge the applicability of the findings to their own settings of interest.

For dependability and confirmability, an external audit was carried out by an external auditor, a doctoral student who was familiar with the methods of naturalistic inquiry and who had full access to the videotapes, audiotapes, transcriptions of interviews and the coding of information.

Results

The questions that were used to guide the study initially form the bases for the analysis of data and discussion of the results.

What are the signs or symbols that are used to communicate in the classroom? How does the instructor of the class interpret those signs? How do students in the class interpret those signs?

Within this Finite Mathematics classroom, signs or symbols were observed that could be categorized in three ways: mathematical vocabulary, disciplinary assumptions, and axioms or theorems. Examples of the symbols and signs used in each of these categorizations as well as the instructor's (Laura) and students interpretations of the symbols used follows.

Mathematical Vocabulary. Within the class mathematical meanings attributed to words formed a vocabulary that students

had to comprehend in order to understand the material. Several of these were evident in the videotaped segments. In any discussion of Markov chains Laura frequently used the term "state."

Me: In this part of the tape you are talking about states. What do you mean by states? What is that?

Laura: Well, when you are talking about Markov chains, you are always doing them within a system. You always have something that is happening and that system is either in one outcome or another outcome and in this case we only had two outcomes... Each of those outcomes becomes a state.

One student stated that she didn't know what a state was. The other eleven students who were interviewed about this segment of the videotape indicated an understanding of the term. Bill was an A- student and Jason, an A student.

Bill: ...I think she was talking about just the fact that the field (goal) kicker either made it, in the state of making it, or in the state of not making it. If that is what she means, I wouldn't use that term, but I think that is what she is talking about. I've kind of let that slide by on the board, I was just trying to see what she was doing on the board.

Jason: They're the different conditions that apply... there are different conditions in the process. Different times, time stages.

While Bill's answer differed from the instructor Laura's in that it was specific to the problem that was presented, Jason's answer was less exact. He seemed to be grasping at random from phrases he heard in class.

Other, lower achieving students in the class were clearer

than Jason in their explanations.

Pati: I think a state is the position that... the condition of the chain before going on to another position or condition... or the next link in the series from one kick to the next kick.

Paola: I think she meant, like kind of a place, ... not even a place, the action in the present, the condition.

While these students' explanations were not as elaborate as the instructor's, not as tied to the actuality of the specific problem as Bill's, they demonstrated an understanding of the term.

Disciplinary Assumptions. An example of a symbol that incorporated a disciplinary assumption was evident when Laura, the instructor, was asked, "what does that transition diagram mean?" Part of her answer, "It is just another way of summarizing the information. ..." provided an example of the kinds of assumptions that those who use mathematics might make about the signs with which they communicate. The transition diagram is sometimes a complex combination of circles, arrows, and numbers, that provide a description of a Markov chain -- a very efficient way of presenting a wealth of information.

Six students were asked what was meant by the transition matrix on the blackboard in the videotaped segment of class. One student (a minority student who came in 24 minutes late to class) said she didn't know. Another student gave a simplistic explanation that was correct but provided no details. Three students gave very literal interpretations of the actual diagram

at which we were looking. One student, with the lowest grade in the class, provided a more elaborate, abstract explanation that indicated a good understanding of what the diagram was meant to represent:

Mitchell: It's a graph of the relationship between two events and the potential linkages between them... the linkages are indicated by the direction arrows and next to each of those is the probability...of going from state one to state two, state one to state one (etc.)...

A second example of the use of symbols in the classroom that incorporated disciplinary assumptions occurred when Laura was solving a general expression (deriving a formula) for a probability matrix.

Me: Why are there letters all over the board? Before there were numbers and decimals that you were adding up and here it just seems messier.

Laura: ...if we did a specific case, all we would be able to do would be to come up with a specific two step matrix. And what we want is a general expression, kind of like a formula for finding it and so we have used general expressions for the probabilities.

Six students were also interviewed about the videotaped segment on solving for a general formula. The best response was given by the Hispanic student who earned a C- for the course:

Paola: She is trying to prove something by using letters that you could substitute numbers for in any case.

Answers from other students ranged from good "I think she is trying to indicate what the numbers represent" to inadequate.

Nanette: ...she's just using two different states, cause she is just showing three people in going from the first state to the same state...

Jason: She is using variables to make it simpler, or to use it as an example, to have it, to show us another way of doing it.

Interestingly, answers given by Nanette, a B+ student, and Jason, an A student, were not nearly as clear as Paola's.

Axioms and Theorems. Some of the explanation that the instructor used relied on axioms or theorems studied earlier in the semester. Probability, a topic that had been covered in an earlier segment of the class, was an important tool in the study of Markov chains. At one point in the interview Laura was asked why she totalled a series of probabilities.

Laura: Because the probability of a set of disjoint outcomes is the sum of each probability.

Some students' answers in this section, while they were not able to quote the axiom, reflected an understanding of the basic concepts.

Terry: I guess to get the total probability of an event happening. And when you have several different combinations you just add the probability of each individual happening.

Mark: We were looking for the probability that he makes the last and those were four situations in which he made the last one, so in order to get the whole situation she needs to add up all the probabilities.

Of the six students who answered this question, the one who scored the highest grade, Bill (A-) and the student who scored lowest, Mark (D-), were not as eloquent.

Bill: Because that is how you get the total

probability, you have to add up each possibility that would satisfy the question you are answering, you have to add up all the probabilities.

Mark: To find the total of the thing she is defining. Every chance that the individual probability or different outcomes that affect the condition of making the last kick... find the probability of that one thing by adding the final probability.

The ability to solve the problems did not seem to rely solely on ability to recall axioms.

Do differences in interpretation of signs in a mathematics classroom vary by success levels of students? Which students more likely to share the instructor's interpretation of those signs?

One assumption of the researcher was that relatively weak students would exhibit gaps in the ways that they talked about mathematics. The expectation was that the higher the achievement level of the student, the more closely that student's explanations and interpretations of the signs used in the class would "match" the instructor's. In some instances, that was true, but in general it was too simplistic a hypothesis.

According to initial analysis there were at least three levels of interpretation emerging from responses in the interview. At the lowest level, there were incomplete, inaccurate, or no explanations about the signs used in response to questions. However, at a second level, students gave "reverbalizations" of what they had heard the instructor say in class. Finally, a higher level of response included answers or

explanations of signs and symbols that involved interpretation and moved beyond echoing the teacher's words. Students were able to provide responses in their own terms.

When students' test scores and final scores were examined there was almost no pattern of response in the above categories. While the students who received the top grades in the class less frequently provided wrong answers to questions, they did not differ from the other students (middle and low grades) in terms of frequency of interpretive answers.

For example, in the above section on "Disciplinary Assumptions," a student who received a C- for the course, Paola, provided the best explanation (and in her own words) for the instructor's derivation of a general formula. The A student, Jason, provided a weaker explanation. Yet, students who understand the usefulness of deriving a general formula have incorporated a disciplinary assumption into their own way of thinking and learning.

Discussion

Tobias (1991) conducted a study that included highly successful academics from the humanities taking calculus and physics classes and highly successful mathematicians and scientists taking humanities classes. She learned that each group of scholars had difficulty with low level college material across disciplinary groupings. Frequently, they reported having trouble organizing information and sorting what was important

from what was unimportant.

One of the most striking findings from the study reported here was the difference in the ways that some high achieving and low achieving students in this mathematics class described the symbols employed in their classroom. Frequently, a low achieving student was able to describe accurately and eloquently the meanings of the symbols used in abstract terms. However, those same students', as indicated by test scores and final grades, were least successful in the classroom. By contrast, frequently the students who earned As in the class gave verbal explanations of symbols employed that were completely inaccurate.

It is possible that those who were high achievers verbally were not high achievers quantitatively and those at the highest level of the mathematics class did not have good verbal skills. This finding in itself was unsurprising - that the high verbal students may be low in quantitative skills. What was surprising was the frequency with which those students could better explain elements of their mathematics course better than high achievers in the same classroom.

Students' responses were categorized according to three subjectively developed categories: those that were wrong, responses that were reverbalizations of what the instructor had said, and interpretive - those that went beyond the instructor's explanation. Those categories seemed unrelated to students' success in the classroom as measured by scores on tests and final grade.

This study raises two questions. First, if students grasp material conceptually and can explain the use of symbols in the abstract, what is it that prevents them from translating that grasp into ability as measured in the classroom? Second, is it important for those who can calculate the answers in a mathematics class to be able to explain the constructs about which they are calculating?

Students in this study had either successfully tested into this Finite Mathematics class or had succeeded in passing lower level prerequisite courses. The range of final grades for the students interviewed was A through D-. However, by the time this last segment of the mathematics class was reached, most of those who were unsuccessful had already withdrawn or failed out of the class. Perhaps those students would be unable to muster a verbal explanation of the symbols being used in class.

Further research might include students from an earlier point in the semester. This would enable the inclusion of a broader range of ability in the student sample. Another study might explore the degree to which students respond to questions in terms of merely describing symbols. That could be contrasted with explanations that include evidence of knowledge about what one does with those symbols.

The results of this research were the beginnings of an attempt to understand and overcome some of the problems with learning processes in college level mathematics classrooms.

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